CERTAIN PROBLEMS RELATED TO THE AERODYNAMICS OF THE GRAVITY FLOW OF A FINELY DISPERSED FREE-FLOWING MATERIAL

V. A. Minko   UDC 541.182.3

We present the results from theoretical and experimental investigations into the determination of the velocities of vertically incident particles for Re < 2000 and for the volumes of air ejected by a stream of a free-flowing material being transferred through closed troughs.

In designing aspiration systems, the greatest difficulties arise in determining the volumes of air entrained (ejected) by the stream of free-flowing material.

This problem was solved analytically for materials made up of heavy pieces [1], in which the coefficient of aerodynamic drag c is assumed to be constant.

Theoretical and experimental investigations into the velocities of fine particles and ejected volumes of air in a transient particle streamlining regime in which c = f(Re) were undertaken to develop a method of calculating the volumes of air ejected by a finely dispersed material.

For the solution of this problem the coefficient of frontal resistance is assumed on the basis of the equation

$$c = 4.1 \mathrm{Re}^{-0.3}\, \Phi. \tag{1}$$

Let us write the equation of motion for a solitary incident fine particle (without consideration of the velocity of the ejected air)

$$m \frac{dV_1}{dt} = gm - \frac{cV_1^2 f \rho_a}{2}. \tag{2}$$

Having substituted the value of c from (1) into (2) and having introduced the notation

$$E = \frac{f\rho_a \cdot 4.1 v^{0.3}}{2mgd^{0.3}} = \frac{0.131 d^{-1.3}}{\rho_m}, \tag{3}$$

we find that

$$\frac{dV_1}{g - EV_1^{1.7}\Phi} = dt. \tag{4}$$

For the solution of (4) we expand the denominator as a Newton binominal [2]. After the expansion and integration, we obtain

$$t = 0.102 V_1 + \frac{5.06 \cdot 10^{-4} V_1^{2.7}\Phi}{\rho_m d^{1.3}} + \frac{4.15 \cdot 10^{-6} V_1^{4.4}\Phi^2}{\rho_m^2 d^{2.6}} + \frac{4 \cdot 10^{-8} V_1^{6.1}\Phi^3}{\rho_m^3 d^{3.9}} + \frac{4.2 \cdot 10^{-10} V_1^{7.8}\Phi^4}{\rho_m^4 d^{5.2}}. \tag{5}$$

The path h traversed by the particle is determined from the expression

$$h = \int_0^t V_1 dt. \tag{6}$$
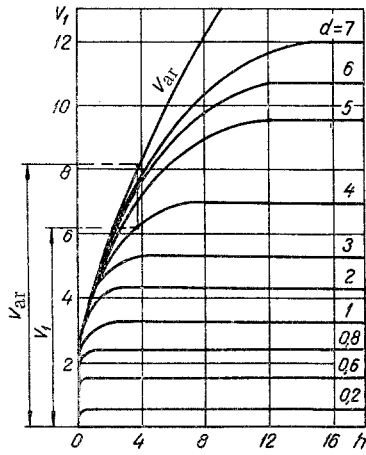
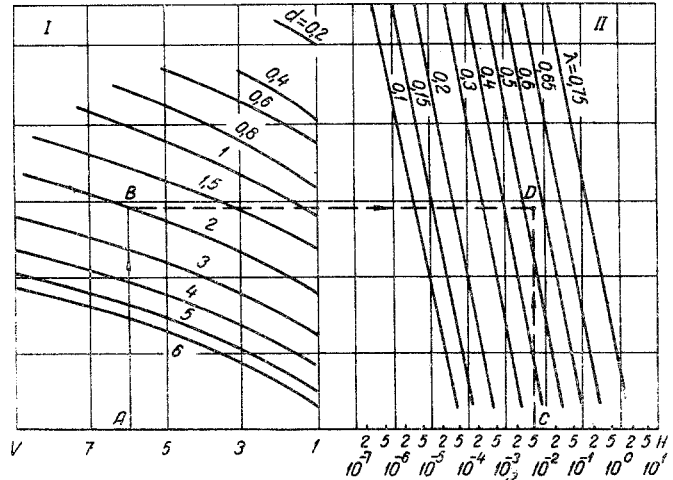Fig. 1.   Fig. 2.

Fig. 1. Nomogram (ρ_m = 3 tons/m^3) to determine the velocity (m/sec) of finely dispersed materials (the numerals with the curves denote the values of d, mm; $V_{ar} = \sqrt{2gh}$).

Fig. 2. Nomogram (V_1 = 6 m/sec, d = 2 mm, H = $5 \cdot 10^{-3}$, key: A $\rightarrow$ B $\rightarrow$ C $\rightarrow$ D, λ = 0.55) for the determination of λ when $V_1 \neq V_{free}$ (the numerals with the curve in (I) denote the values of d, mm, while those with the curves in (2) denote the values of λ).

After solution of (6) we finally obtain

$$h = 0.051\,V_1^2 + 3.37 \cdot 10^{-4}\,\frac{d^{-1.3}V_1^{3.7}\varPhi}{\rho_m} + 2.85 \cdot 10^{-6}\,\frac{d^{-2.6}\,V_1^{5.4}\varPhi^2}{\rho_m^2}$$

$$+\,2.6 \cdot 10^{-8}\,\frac{d^{-3.9}V_1^{7.1}\varPhi^3}{\rho_m^3} + 2.58 \cdot 10^{-10}\,\frac{d^{-5.2}\,V_1^{8.8}\varPhi^4}{\rho_m^4} = 0.051\,V_{ar}^2 \tag{7}$$

With (7) we can determine the velocity V_1 for a single particle with consideration of the drag developed by the air. In general form $V_1 = \psi$ (d, ρ_m, V_{ar}).

We compiled nomograms to make the calculation more convenient; Fig. 1 shows the nomogram for ρ_m = 3 m/m^3.

In determining the air volumes ejected by the vertical stream of finely dispersed materials, we adopted a model of the phenomenon proposed in [3], where the onset of motion for the air is treated as a consequence of overcoming the frontal resistance of the air by the particles of the material.

Let us compile the balance of the energy expended on the development of air velocity:

$$dT = \frac{6 \cdot 4.1\, Re^{-0.3}\varPhi G}{\pi d^{1.3}\,\rho_m} \left[\frac{\pi d^2 \rho_a\,(V_1 - \lambda V_1)^2}{8.} \right] V_1 dt$$

$$= \frac{6 \cdot 1.22 \cdot \varPhi \cdot G \cdot 4.1\,V_1^{-0.3}\,(1-\lambda)^{-0.3}\,d^{-0.3}\,V_1^3(1-\lambda)^2}{8\rho_m\,dv^{-0.3}}\,dt = \frac{3.75\,G\varPhi V_1^{2.7}\,(1-\lambda)^{1.7}}{\rho_m\,d^{1.3}\,v^{-0.3}}\,dt, \tag{8}$$

$$dT = \frac{MV_1^{2.7}\,(1-\lambda)^{1.7}}{d^{1.3}}\,dt, \tag{9}$$

where

$$M = \frac{3.75\,G\varPhi}{\rho_m\,v^{-0.3}}, \tag{10}$$

$$\lambda = \frac{V_a}{V_1}. \tag{11}$$

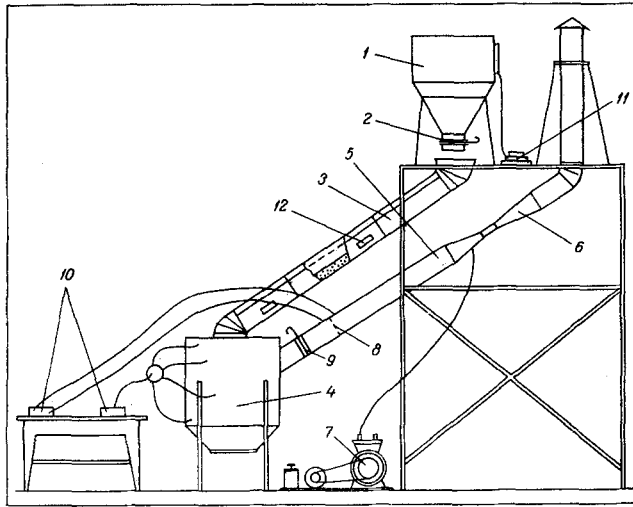Two cases are possible in the motion of finely dispersed materials in troughs.

Fig. 3. Test stand to study the ejection properties of materials: 1) bunker; 2) slide valve; 3) trough; 4) hermetically sealed bunker; 5) conduit; 6) ejector; 7) compressor; 8) diaphragm; 9) air valve; 10) micromanometers; 11) vibrator; 12) viewing window.
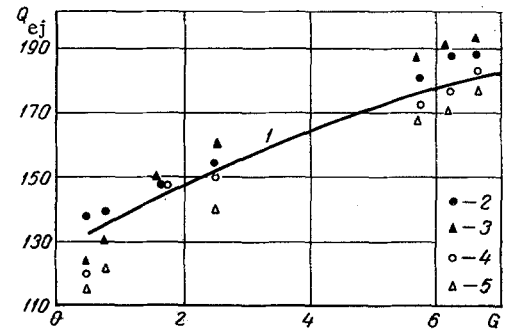
Fig. 4. Comparison of analytical and experimental air volumes for d = 0.44 mm, F = 0.0132 m², R = 7200 N · sec²/m⁸: 1) analytical curve; 2) α = 90°· 3) 64; 4) 57; 5) 54; Q_{ej}, m³/h; G, tons/h.

Case I. The velocity of motion for the particles of the material reached its limit value, i.e., V_1 = V_{free}. Here we assume that λ will be constant throughout the entire height of the trough. Then, integrating (9), we obtain

$$T = \frac{MV_1^{2,7}(1-\lambda)^{1,7}}{d^{1,3}}t + C.$$ (12)

Since for t = 0, T = 0, we have C = 0. Having substituted the value of t from (5) into (12) and bearing in mind that

$$T = RQ_{ej}^3 = R\lambda^3 V_1^3 F^3,$$ (13)

we find

$$RQ_{ej}^3 = \frac{MV_1^{2,7}(1-\lambda)^{1,7}}{d^{1,3}}\left[0,102\,V_4 + 3.85\cdot10^{-3}\frac{0.131\,d^{-1,3}}{\rho_m}\varPhi V_1^{2,7} + 2.42\cdot10^{-4}\left(\frac{0.131\,d^{-1,3}}{\rho_m}\right)^2\varPhi^2 V_1^{4,4}\right.$$

$$\left. + 1.77\cdot10^{-5}\left(\frac{0,131\,d^{-1,3}}{\rho_m}\right)^3\varPhi^3 V_j^{6,1} + 1.41\cdot10^{-6}\left(\frac{0,131\,d^{-1,3}}{\rho_m}\right)^4\varPhi^4 V_1^{7,8}\right] = 0.102\,V_4$$

$$ + \frac{5.06\cdot10^{-4}\varPhi V_1^{2,7}}{d^{1,3}\rho_m} + \frac{4.15\cdot10^{-6}\varPhi^2 V_1^{4,4}}{d^{2,6}\rho_m^2} + \frac{4\cdot10^{-8}\varPhi^3 V_1^{6,1}}{d^{3,9}\rho_m^3} + \frac{4.2\cdot10^{-10}\varPhi^4 V_1^{7,8}}{d^{5,2}\rho_m^4}.$$ (14)

In (13) R is the hydraulic characteristic of the trough and is determined from the formula

$$R^* = \frac{\Sigma\zeta\rho_a}{2F^2}.$$ (15)

Having substituted (13) into (14),

$$\frac{\lambda^3}{(1-\lambda)^{1,7}} = \frac{3,75\,G\varPhi}{\rho_m v^{-0,3} d^{1,3} RF^3 V_1^{0,3}}t.$$ (16)

We denote

$$\frac{3,75\,G\varPhi}{v^{-0,3}\rho_m RF^3} = \frac{0,13\,G\varPhi}{\rho_m RF^3} = H,$$ (17)

---

* $R = \dfrac{h_1}{Q_{ej}^2}$, $h_1 = \Sigma\zeta\,\dfrac{\rho_a V_a^2}{2}$, $Q_{ej}^2 = F^2 V_a^2$, i.e., $R = \dfrac{\rho_a}{2}\,\dfrac{\Sigma\zeta}{F^2}$.

$$\frac{t}{d^{1,3}V^{0,3}} = N. \tag{18}$$

Then

$$\frac{\lambda^3}{(1-\lambda)^{1,7}} = HN. \tag{19}$$

In general form

$$\lambda = f_1(H, N) = \psi_1(H, d, V_1, \rho_m, \Phi).$$

For convenience of calculation we compiled nomograms based on (19) for a coefficient of the form Φ = 3.49 [4].

Case II. The particle velocity V_1 did not reach its maximum value, i.e., $V_1 \neq V_{free}$. In this case the coefficient λ is a variable. After solution of the differential equation (9) and simplifications, we derived an equation for λ, i.e.,

$$\frac{\lambda^3}{(1-2,28\lambda+1,28\lambda^2)} = \frac{H \cdot 2,8 \cdot 10^{-2} V_1^{0,7}}{d^{1,3}}. \tag{20}$$

As we can see from (20), here λ is indepenendent of material density ρ_m.

For convenience of calculation, according to (20), we compiled the nomogram shown in Fig. 2:

$$\lambda = f_2(d, V_1, H).$$

To test the analytical equations (19) and (20), by means of which we can determine the air volumes ejected by the stream of finely dispersed material, we undertook experimental studies on a special test stand (Fig. 3).

The investigations were carried out on a material in which particle size was in the range d_{av} = 14.1–0.15 mm and for various regimes (the angle of trough inclination was varied, as was the cross section of the trough, and the height of the material drop).

The stand consists of bunker 1 into which the test material is charged. The material moves along trough 3 into hermetically sealed bunker 4. The ejected air is withdrawn along conduit 5 by means of pneumatic ejector 6. The supply of air for the ejector comes from compressor 7.

The quantity of air passing through conduit 5 is measured on the basis of the pressure difference across a precalibrated diaphragm 8.

The volume of air withdrawn from bunker 4 is regulated by slide valve 9. The uniformity of material outflow is ensured by operation of vibrator 11.

In performing these tests on the stand to determine the volumes of ejected air, we used slide valve 2 to pass the material into the trough, and slide valve 9 assumed a position such that the rarefaction in the bunker amounts to ± 0 N/m^2. At precisely this moment the quantity of the withdrawn air is measured on the basis of the pressure difference across the micromanometer.

Since the bunker, the air conduit, and the trough are hermetically sealed, the quantity of air entrained with the material is equal to the volume of air removed from bunker 4 (with a rarefaction of ± 0 N/m^2 in the bunker).

The tests were carried out on a specially prepared ore with d_{av} = 14.1 mm, 7.06, 2.5, 0.89, 0.44, 0.21 mm, and d < 0.15 mm. It was established that for the analytical determination of the ejected-air volumes of a material with d_{av} = 14.1 and 7.06 mm it is necessary to use the earlier developed method for materials consisting of large pieces [1], where the coefficient of frontal resistance is assumed to be constant (the self-similarity region). For a material with 0.2 mm $\leq d_{av} \leq$ 2.5 mm the actual volumes of air are in satisfactory agreement with the analytical volumes, determined from (19) and (20) for trough inclination angles of α = 90°, 64, 54. With an inclination angle of α = 48° the actual volumes differed from the analytical volumes by 70-80%, which can be explained by the presence of a layer of material and, correspondingly, by a change in the regime of streamlining the particles with air.

Figure 4 shows the curve for the analytical volumes and the experimental points for various angles of inclination for $d_{av} = 0.44$ mm; $F = 0.0132$ m^2, and $R = 7200$ N $\cdot$ sec^2/m^8. The experimental volumes were obtained by averaging 10-12 measurements, and the confidence coefficient was 0.92-0.95. Comparison of the experimental and analytical air volumes (Fig. 4) shows satisfactory agreement for the results (for a standard deviation of 20% the confidence coefficient is 0.91-0.99 [5]).

With a material particles diameter of $d < 0.15$ mm the actual volumes of air were substantially greater than the theoretical, which can be explained by the fact that the motion of the powder-like materials proceeds in "packets" of particles.

These results thus make it possible to recommend (19) and (20) to determine the volumes of air ejected by the gravity flow of the granular material (0.2 mm $\leq d_{av} \leq 2.5$ mm).

NOTATION

c	is the coefficient of frontal drag;
V_1	is the velocity of material particle motion, with consideration of air drag, m/sec;
Φ	is the dynamic shape factor for the particle;
ν	is the kinematic coefficient of viscosity for the air, m^2/sec;
d	is the particle diameter, m;
f	is the midsection, m^2;
ρ_m	is the material density, kg/m^3;
h	is the load height, m;
V_{ar}	is the velocity of particle motion without consideration of the resistance of the air medium, m/sec;
ρ_a	is the air density, kg/m^3;
T	is the kinematic energy expended on the development of the ejection air flow, N $\cdot$m/sec;
G	is the material flow rate, kg/sec;
V_a	is the velocity of the ejected air, m/sec;
R	is the trough resistance characteristic, N $\cdot$ sec^2/m^8;
V_{free}	is the free-fall velocity for the particle, m/sec;
λ	is the coefficient by means of which we take into consideration the lag in the air velocity from the velocity of the material.

LITERATURE CITED

1. V. V. Nedin and O. D. Neykov, Combating Dust in Mines [in Russian], Nedra, Moscow (1965).
2. V. I. Smirnov, A Course in Higher Mathematics [in Russian], Vol. 1, Moscow (1958).
3. S. E. Butakov, The Aerodynamics of Industrial Ventilation Systems [in Russian], Profizdat, Moscow (1949).
4. B. N. Lobaev, Design of Air Conduits [in Russian], UkrSSR, Kiev (1959).
5. A. N. Zaydel', Elementary Estimates of Measurement Errors [in Russian], Nauka, Moscow—Leningrad (1965).